Velocity of simple Harmonic oscillator

The displacement of a simple harmonic oscillator at any instant of time t is given by

$$Y = a \sin(\omega t + \phi) \quad \text{——— (i)}$$

The velocity is defined as the time rate of change of displacement.

$$\therefore \text{velocity} \quad V = \frac{dy}{dt} = \dot{Y} = a\omega \cos(\omega t + \phi)$$

$$= a\omega \sin\left[\omega t + \phi + \frac{\pi}{2}\right] \quad \text{——— (ii)}$$

As $\sin(\omega t + \phi) = \frac{Y}{a}$, $\cos(\omega t + \phi) = \sqrt{1 - \frac{Y^2}{a^2}}$

$$= \frac{\sqrt{a^2 - Y^2}}{a}$$

$$\therefore \quad V = \omega\sqrt{a^2 - Y^2} \quad \text{——— (iii)}$$

Maximum velocity :- The velocity of the oscillator is maximum when $\sin\left(\omega t + \phi + \frac{\pi}{2}\right) = 1$

$$\therefore V_{max} = a\omega$$

The value of $V = V_{max}$ when $Y = 0$ i.e, the particle executing S.H.M is in its mean position comparing equation (i) and (ii), we find that the velocity of S.H.M at any instant of time t leads the displacement by a phase difference $\pi/2$ radian (or 90°) i.e, the two are in quadrature.

The velocity varies harmonically with the same frequency ω.

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